A two–slit experiment which distinguishes between the standard and Bohmian quantum mechanics

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Abstract

In this investigation, we have suggested a special two-slit experiment which can distinguish between standard and Bohmian quantum mechanics, even at the statistical level. At the first step, we have shown that observable individual predictions at suitable time intervals, obtained from these theories, are inconsistent. But, at the statistical level, they are consistent as was expected. Then, using suitable arrangements, we have shown that not only observable disagreement between the two theories exists at the individual level, but that using selective detection, there are novel observable predictions that either standard quantum mechanics is silent about them or that its predictions are in disagreement with those of Bohmian mechanics at the statistical level. Finally, we have examined suitable conditions for performing such experiment.

Keywords: Bohmian quantum mechanics, Two–slit experiment, Selective detection, Statistical disagreement

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1 Introduction

Since the standard quantum mechanics (SQM) and Bohmian quantum mechanics (BQM) have similar sets of equations, it seems that these two must be empirically equivalent. Bohm and his collaborators believed that their theory will, in every conceivable experiment, yield the same observable results as SQM [1–4]. Bohm, himself, in responding to the question of whether there is any new prediction by his theory, said (1986): "Not the way it's done. There are no new predictions because it is a new interpretation to the same theory" [4]. In fact, when Bohm presented his theory in 1952, experiments could be done with an almost continuous beam of particles, but not with individual particles. Thus, Bohm cooked his theory in such a fashion that it would be impossible to distinguish his theory from SQM. For this reason, when J. Bell [5] talked about the empirical equivalence of the two theories, he was more cautious: "It [the de Broglie-Bohm version of non–relativistic quantum mechanics] is experimentally equivalent to the usual version in so far as the latter is unambiguous". Thus the question arises as to whether there are phenomena which are well–defined in one theory (due to the presence of path for particles) but ambiguous in the other one or phenomena which have different observable results in the two theories? At first it seems that the transition of a quantum system through a potential barrier provides a good case. Here, there is no well defined transit time between the two ends of the barrier in the SQM, because time is considered to be a parameter and not a dynamical variable having a corresponding Hermitian operator [6]. For BQM, however, the passage of a particle between any two points is conceptually well defined. But, the recent work of Abolhasani and Golshani [7] indicates that it is not practically feasible to use this experiment to distinguish between these two theories. In addition, there have been other recent reports suggesting the incompatibility of these two theories [8]. But, Marchildon [9] has argued that this claim is unfounded. On the other hand, Dewdney, Hardy and Squires [10], carried out a detailed calculation on a Gedanken experiment and showed that a quantum particle can excite a detector while passing far away from it, if one interprets the Bohmian trajectory as representing the real particle position. Then, SQM and BQM are in complete disagreement with each other. The same Gedanken experiment was also noted by Bell [5]. It is worthy to note that Griffiths [11] recently investigated this subject by his consistent histories approach and compared his results with those of SQM and BQM. Aharonov et al. [12] considered another thought experiment. Their conclusion, as well as Griffiths', was that the formally introduced Bohmian trajectories are just mathematical constructions with no relation to the actual motion of the particle. Furthermore, Ghose [13] has recently claimed that by devising a new version of the two slit experiment, one can distinguish between the two theories. But, in these works BQM yields the same statistical results for particle positions as does SQM. Although this latter incompatibility is also rejected by Marchildon, we will see that his argument is imperfect and that Ghose's work is a special case of our extended results.

Here we have shown that in a specific double–slit experiment, using Gaussian wave functions representing two non–relativistic bosonic particles–with symmetric wave functions and symmetric experimental arrangement, the predictions of BQM are in complete disagreement with SQM at the individual level, but at the statistical level they yield the same results, as was expected. Furthermore, we show that under suitable experimental arrangements and using selective detection, BQM can predict results which not only show differences between the two theories in the detection of particles in suitable time intervals at the individual level, but they also bring in the possibility of novel predictions at the statistical level, which are different from those of SQM, or predictions that SQM is silent about them. In addition, this experiment can provide a test for the question of whether the concept of position introduced by Bohm is a real one or not.

2 A review of Bohmian mechanics

Here we give a short review of Bohmian mechanics and consider the problem of its equivalence with the standard quantum mechanics. We consider n particles with masses $m_1, m_2..., m_n$ and coordinates $\overrightarrow{x_1}, \overrightarrow{x_2}, ..., \overrightarrow{x_n}$. Writing the Schrödinger wave function in the form

$$\psi(\overrightarrow{x_1}, \overrightarrow{x_2}, ..., \overrightarrow{x_n}; t) = R(\overrightarrow{x_1}, \overrightarrow{x_2}, ..., \overrightarrow{x_n}; t) e^{iS(\overrightarrow{x_1}, \overrightarrow{x_2}, ..., \overrightarrow{x_n}; t)/\hbar}, \tag{1}$$

the path of the i th particle is obtained from the following first order differential equation [1–3]:

$$\dot{x}_i(\overrightarrow{x}, t) = \frac{1}{m_i} \nabla_i S(\overrightarrow{x}, t) = \frac{\hbar}{m_i} Im \left(\frac{\nabla_i \psi(x, t)}{\psi(x, t)} \right), \tag{2}$$

where $\overrightarrow{x} = (\overrightarrow{x_1}, \overrightarrow{x_2}, ..., \overrightarrow{x_n})$. Replacing (1) into the Schrödinger equation

$$i\hbar \frac{\partial \psi(\overrightarrow{x}, t)}{\partial t} = \left[\sum_{i=1}^{n} \frac{-\hbar^2}{2m_i} \nabla_i^2 + V(\overrightarrow{x}, t) \right] \psi(\overrightarrow{x}, t), \tag{3}$$

leads to

$$\frac{\partial S(\overrightarrow{x}, t)}{\partial t} + \sum_{i=1}^{n} \frac{(\nabla_i S(\overrightarrow{x}, t))^2}{2m_i} + Q(\overrightarrow{x}, t) + V(\overrightarrow{x}, t) = 0, \tag{4}$$

and

$$\frac{\partial R^2(\overrightarrow{x},t)}{\partial t}+\sum_{i=1}^{n}\nabla_i.\left(R^2(\overrightarrow{x},t)\frac{\nabla_i S(\overrightarrow{x},t)}{m_i}\right)=0, \tag{5}$$

where

$$Q(\overrightarrow{x},t)=\sum_{i=1}^{n}\frac{-\hbar^2}{2m_i}\frac{\nabla_i^2 R(\overrightarrow{x},t)}{R(\overrightarrow{x},t)}, \tag{6}$$

is the so–called quantum potential of the system of n particles. Equations (2) and (3) yield a consistent theory. From (5) one can see that R^2 is a conserved quantity. It is sufficient to assume that at $t=0$ the distribution of particles is given by

$$P(\overrightarrow{x})=R^2(\overrightarrow{x})=\mid\psi(\overrightarrow{x})\mid^2 . \tag{7}$$

Then, using the continuity equation, one can show that this equality holds at other times and that the statistical predictions of the two theories are the same. Furthermore, if $\widehat{A}=\widehat{A}(\overrightarrow{x},\overrightarrow{p})$ is considered to represent a Hermitian operator, and we define

$$A(\overrightarrow{x},t)=Re\frac{\psi^*(\overrightarrow{x},t)\widehat{A}\psi(\overrightarrow{x},t)}{\psi^*(\overrightarrow{x},t)\psi(\overrightarrow{x},t)}, \tag{8}$$

as representing a local expectation value, to be identified with a property of a particle or ensemble of particles, then the calculation of the expectation values in SQM will always be equivalent to averaging over an ensemble of particles in BQM. This is because we have

$$\langle A\rangle=\int R^2(\overrightarrow{x},t)A(\overrightarrow{x},t)d^3x=\int\psi^*(\overrightarrow{x},t)\widehat{A}\psi(\overrightarrow{x},t)d^3x=\langle\widehat{A}\rangle. \tag{9}$$

Thus BQM is constructed in such a way that its observational results, at the statistical level, are consistent with those of SQM. Here, we shall see that if, in an ensemble of particles, the paths of individual particles lacks significance, i.e., the particles are considered to be non-distinguishable, then the predictions of the two theories are consistent. But, if the history of the particles affects their detection, then we can expect to have different results for the two theories, even at the statistical level.

3 A double-slit experiment to distinguish between SQM and BQM

We consider the following experiment. A pair of identical non-relativistic bosonic particles originate simultaneously from a point source S_1. We assume that the intensity of the beam is so low that a time we have only a single pair of particles passing through the slits. Since the direction of the emission of each particle can be considered to be random, we assume that the detection screen S_2 registers only those pairs of particles that reach it simultaneously. Then, the interference effects of single particles are eliminated. Furthermore, it is assumed that the detection process has no causal role in the phenomenon of interference [3]. In the coordinate system (x, y), with the origin at O, the centers of the two slits are located at $(0, \pm Y)$. Figure 1 shows schematic arrangement of this two-slit experiment. We take the incident wave to be a plane wave of the form

$$\psi_{in}(x_1, y_1; x_2, y_2; t) = ae^{i[k_x(x_1+x_2)+k_y(y_1+y_2)]}e^{-iEt/\hbar}, \tag{10}$$

where a is a constant and $E = E_1 + E_2 = \hbar^2(k_x^2 + k_y^2)/m$ is the total energy of the system of two particles. For mathematical simplicity we avoid slits with sharp edges which produce mathematical complexity of Fresnel diffraction, i.e., we assume that the slits have soft edges, so that the Gaussian wave packets are produced along the y-direction, and that the plane wave along the x-axis remain unchanged [3]. In fact, the one-particle wave function should be represented by Gaussian wave packets rather than plane or spherical waves as utilized by Ghose [13] and Marchildon [9] respectively. We take the time of the formation of the Gaussian wave to be $t = 0$. Then, the emerging wave packets from the slits A and B are respectively

$$\psi_A(x, y) = a(2\pi\sigma_0^2)^{-1/4}e^{-(y-Y)^2/4\sigma_0^2}e^{i[k_x x+k_y(y-Y)]}, \tag{11}$$

$$\psi_B(x, y) = a(2\pi\sigma_0^2)^{-1/4}e^{-(y+Y)^2/4\sigma_0^2}e^{i[k_x x-k_y(y+Y)]}, \tag{12}$$

where σ_0 is the half-width of each slit.

Now, for this two–particle system, the total wave function at the detection screen S_2, at time t, is

$$\begin{aligned}
&\psi(x_1, y_1; x_2, y_2; t) = \\
&\quad N[\psi_A(x_1, y_1, t)\psi_B(x_2, y_2, t) + \psi_A(x_2, y_2, t)\psi_B(x_1, y_1, t) \\
&\quad +\psi_A(x_1, y_1, t)\psi_A(x_2, y_2, t) + \psi_B(x_1, y_1, t)\psi_B(x_2, y_2, t)],
\end{aligned} \tag{13}$$

with

$$\psi_A(x, y, t) = a(2\pi\sigma_t^2)^{-1/4}e^{-(y-Y-u_yt)^2/4\sigma_0\sigma_t}e^{i[k_x x+k_y(y-Y-u_yt/2)-E_xt/\hbar]}, \tag{14}$$

$$\psi_B(x, y, t) = a(2\pi\sigma_t^2)^{-1/4}e^{-(y+Y+u_yt)^2/4\sigma_0\sigma_t}e^{i[k_x x-k_y(y+Y+u_yt/2)-E_xt/\hbar]}, \tag{15}$$

where N is a reparameterization constant and

$$\sigma_t = \sigma_0(1 + \frac{i\hbar t}{2m\sigma_0^2}), \tag{16}$$

$$u_y = \frac{\hbar k_y}{m}; E_x = \frac{1}{2}mu_x^2, \tag{17}$$

where u_x and u_y are initial group velocities corresponding to each particle in the x and y directions respectively. Note that because of the symmetry of the wave function $\psi(x_1, y_1; x_2, y_2; t)$, the particles 1 and 2 are indistinguishable in SQM.

It is well-known from SQM that the probability of simultaneous detection of the particles at y_M and y_N, at the screen S_2, located at $x_1 = x_2 = D$, at $t = D/u_x$ is equal to

$$P_{12}(y_M, y_N) = \int_{y_M}^{y_M+\triangle} dy_1 \int_{y_N}^{y_N+\triangle} dy_2 |\psi(x_1, y_1; x_2, y_2; t)|^2. \tag{18}$$

The parameter Δ, which is taken to be small, is a measure of the size of the detectors. We shall see that this prediction of SQM differs from that of BQM.

4 The predictions of BQM for the suggested experiment

In BQM, the complete description of the system is given by specifying the location of the particles, in addition to their wave function which has the role of guiding the particles according to (2). Thus the path of particles distinguishes them, and each one of them can be studied separately. Here, the speed of the particles 1 and 2 in the direction y is given, respectively, by

$$\dot{y}_1(x_1, y_1; x_2, y_2; t) = \frac{\hbar}{m} Im \frac{\partial_{y_1}\psi(x_1, y_1; x_2, y_2; t)}{\psi(x_1, y_1; x_2, y_2; t)}, \tag{19}$$

$$\dot{y}_2(x_1, y_1; x_2, y_2; t) = \frac{\hbar}{m} Im \frac{\partial_{y_2}\psi(x_1, y_1; x_2, y_2; t)}{\psi(x_1, y_1; x_2, y_2; t)}. \tag{20}$$

With the replacement of $\psi(x_1, y_1; x_2, y_2; t)$from (13), we have

$$\begin{aligned}
\dot{y}_1 &= \frac{\hbar}{m} Im\{\frac{1}{\psi}[[-2(y_1 - Y - u_y t)/4\sigma_0\sigma_t + ik_y]\psi_{A_1}\psi_{B_2} \\
&+ [-2(y_1 + Y + u_y t)/4\sigma_0\sigma_t - ik_y]\psi_{A_2}\psi_{B_1} \\
&+ [-2(y_1 - Y - u_y t)/4\sigma_0\sigma_t + ik_y]\psi_{A_1}\psi_{A_2} \\
&+ [-2(y_1 + Y + u_y t)/4\sigma_0\sigma_t - ik_y]\psi_{B_1}\psi_{B_2}]\},
\end{aligned} \tag{21}$$

$$\dot{y}_2 = \frac{\hbar}{m} Im\{\frac{1}{\psi}[[-2(y_2 + Y + u_y t)/4\sigma_0\sigma_t - ik_y]\psi_{A_1}\psi_{B_2}$$

$$
\begin{aligned}
&+ \quad [-2(y_2 - Y - u_y t)/4\sigma_0\sigma_t + ik_y]\psi_{A_2}\psi_{B_1} \\
&+ \quad [-2(y_2 - Y - u_y t)/4\sigma_0\sigma_t + ik_y]\psi_{A_1}\psi_{A_2} \\
&+ \quad [-2(y_2 + Y + u_y t)/4\sigma_0\sigma_t - ik_y]\psi_{B_1}\psi_{B_2}]\}.
\end{aligned} \tag{22}
$$

On the other hand, from (14) and (15) one can see that

$$
\begin{aligned}
\psi_A(x_1, y_1, t) &= \psi_B(x_1, -y_1, t), \\
\psi_A(x_2, y_2, t) &= \psi_B(x_2, -y_2, t),
\end{aligned} \tag{23}
$$

which indicates the reflection symmetry of $\psi(x_1, y_1; x_2, y_2; t)$ with respect to the x–axis. Using this symmetry in (21) and (22) we have

$$
\begin{aligned}
\dot{y}_1(x_1, y_1; x_2, y_2; t) &= -\dot{y}_1(x_1, -y_1; x_2, -y_2; t), \\
\dot{y}_2(x_1, y_1; x_2, y_2; t) &= -\dot{y}_2(x_1, -y_1; x_2, -y_2; t).
\end{aligned} \tag{24}
$$

These relations show that if $y_1(t) = y_2(t) = 0$, then the speed of each particles along the y axis is zero along the symmetry axis x. This means that none of the particles can cross the x–axis nor is tangent to it. This conclusion is the result of combining bosonic and geometrical symmetries. In fact, if we had not considered the bosonic symmetry, the two particle–wave function $\psi(x_1, y_1; x_2, y_2; t)$ would have not been symmetric under the reflection with respect to the x–axis. The fact that the paths of the two particles are located on the two sides of the x–axis could lead, under suitable conditions, to a discrepancy between the predictions of SQM and BQM, particularly at the statistical level. If we consider $y = (y_1 + y_2)/2$ to be the vertical coordinate of the centre of mass of the two particles, then we can write

$$
\begin{aligned}
\dot{y} &= (\dot{y}_1 + \dot{y}_2)/2 \\
&= \frac{\hbar}{2m} Im\{\frac{1}{\psi}[(-\frac{y_1 + y_2}{2\sigma_0\sigma_t})(\psi_{A_1}\psi_{B_2} + \psi_{A_2}\psi_{B_1} + \psi_{A_1}\psi_{A_2} + \psi_{B_1}\psi_{B_2}) \\
&+ (\frac{Y + u_y t}{\sigma_0\sigma_t} + 2ik_y)(\psi_{A_1}\psi_{A_2} - \psi_{B_1}\psi_{B_2})]\} \\
&= \frac{(\hbar/2m\sigma_0^2)^2 t(y_1 + y_2)/2}{1 + (\hbar/2m\sigma_0^2)^2 t^2} + \frac{\hbar}{2m} Im\frac{1}{\psi}(\frac{Y + u_y t}{\sigma_0\sigma_t})(\psi_{A_1}\psi_{A_2} - \psi_{B_1}\psi_{B_2}).
\end{aligned} \tag{25}
$$

Now, we consider the following two special cases:

(1) Each particle passes through one of the slits. Then using the symmetry of the problem, we can write

$$
\psi_{A_1} = \psi_{B_2}; \psi_{A_2} = \psi_{B_1}. \tag{26}
$$

In this case, the equation of motion for the y coordinate of the centre of mass (25) is simplified to

$$
\dot{y} = \frac{(\hbar/2m\sigma_0^2)^2}{1 + (\hbar/2m\sigma_0^2)^2 t^2} y t. \tag{27}
$$

Had we neglected the last two terms of (13) as was done in [13] we would have obtained the same result. The significance of these two terms, however, will become apparent shortly when we consider selective detection. Solving the differential equation (27), we get the path of the y coordinate of the centre of mass

$$y = y_0\sqrt{1 + (\hbar/2m\sigma_0^2)^2 t^2}. \tag{28}$$

If at $t = 0$ the centre of mass of the system is exactly on the x–axis, then $y_0 = 0$, and centre of mass of the system will always remain on the x–axis. Thus, the two particles will be detected at points symmetric with respect to the x–axis. This differs from the prediction of SQM, as the probability relation (18) shows. Figure 1 shows one of the typical inconsistencies which can be predicted. In practice y_0 could differ from zero but be very small. But, if $\hbar t/2m\sigma_0^2 \ll 1$, we still detect the particles symmetrical with respect to the x–axis, to a good approximation. For example, if $\sigma_0 = 10^{-7}m$ and $t_{min} = D/(u_x)_{max} \sim 0.3/0.1c = 10^{-8}s$, then the condition for the symmetrical detection of the particles , with $y \simeq 0$, is

$$m \gg \frac{\hbar t}{2\sigma_0^2} \geq \frac{10\hbar D}{2\sigma_0^2 c} \simeq 30MeV. \tag{29}$$

For instance, if we could use sources which emit pairs of $K^\pm$ mesons simultaneously, with $M_{K^\pm} = 493.6MeV$ and the mean life time $\tau = 1.2 \times 10^{-8}s$, the possibility of securing the aforementioned case is provided. Of course, if $y_0 \neq 0$, but the condition $\hbar t/2m\sigma_0^2 \ll 1$ is not satisfied , then the x–axis will not be an axis of symmetry and we need to detect a pair of particles on the two sides of the x–axis to determine the new y. All other pairs will be detected symmetrically with respect to this new y, and again there is going to be a discrepancy between the SQM and BQM for suitable time intervals, at the individual level (later on, we shall show that the same is true even at the statistical level). We return to this condition later.

(2) Both particles pass through the same slit. In this case we have

$$\psi_{A_1} = \psi_{A_2}; \psi_{B_1} = \psi_{B_2}. \tag{30}$$

Using this relation in (21), we get

$$\begin{aligned} \dot{y}_1 &= \frac{\hbar}{m} Im \frac{1}{\psi}\{(\frac{-y_1}{\sigma_0\sigma_t})\psi_{A_1}\psi_{B_2} + (\frac{-y_1}{2\sigma_0\sigma_t})(\psi_{A_1}\psi_{A_2} + \psi_{B_1}\psi_{B_2}) \\ &+ (\frac{Y + u_y t}{2\sigma_0\sigma_t} + ik_y)(\psi_{A_1}\psi_{A_2} - \psi_{B_1}\psi_{B_2})\} \\ &= \frac{(\hbar/2m\sigma_0^2)^2 y_1 t}{1 + (\hbar/2m\sigma_0^2)^2 t^2} + \frac{\hbar}{2m} Im \frac{1}{\psi}(\frac{Y + u_y t}{\sigma_0\sigma_t} + 2ik_y)(\psi_{A_1}\psi_{A_2} - \psi_{B_1}\psi_{B_2}). \end{aligned} \tag{31}$$

A similar relation is obtained for $\dot{y}_2$. It suffices to change the indices 1 to 2. Replacing $\dot{y}_1$ or $\dot{y}_2$ in (25), or calculating $\dot{y}_2 - \dot{y}_1$ directly, we get

$$\dot{y}_2 - \dot{y}_1 = \frac{(\hbar/2m\sigma_0^2)^2}{1 + (\hbar/2m\sigma_0^2)^2 t^2}(y_2 - y_1)t. \tag{32}$$

If $\epsilon(t) = y_2 - y_1$ represents the distance of the particles along the y–axis, then solving the differential equation (32), we get

$$\epsilon = \epsilon_0 \sqrt{1 + (\hbar/2m\sigma_0^2)^2 t^2}. \tag{33}$$

Using this equation and the equation (28), we get a time–independent relation

$$\frac{\epsilon}{y} = \frac{\epsilon_0}{y_0}. \tag{34}$$

It seems possible to determine ϵ and y through the detection process. In addition, since we have $\epsilon_0 \leq \sigma_0$, thus the detectable maximum separation of the two particles on one side of the x–axis, after a long time, is

$$\epsilon_{max} = \frac{\hbar t}{2m\sigma_0}. \tag{35}$$

So far we have been dealing with the difference between the SQM and BQM in the detection of pairs of particles on the two sides of x–axis at the individual level. Now, the question arises as to whether this difference persists if we deal with an ensemble of pairs of particles? To find the answer to this question, we consider an ensemble of pairs of particles that have arrived at the detection screen S_2 at different times t_i. The probability of simultaneous detection for all pairs of particles arriving at S_2 is

$$\begin{aligned} P_{12} &= \sum_{i=1}^{\infty} \frac{1}{\delta(0)} \int dy_1 \int dy_2 P(y_1, y_2, t) \\ &\times [\delta(y_1(t_i) + y_2(t_i) - y_0\sqrt{1 + (\hbar/2m\sigma_0^2)^2 t^2}) \\ &+ \delta(y_2(t_i) - y_1(t_i)) - \epsilon_0\sqrt{1 + (\hbar/2m\sigma_0^2)^2 t^2})] \\ &\times \delta(y_1 - y_1(t_i))\delta(y_2 - y_2(t_i)) \\ &= \sum_{i=1}^{\infty} [P(y_1(t_i), y_0\sqrt{1 + (\hbar/2m\sigma_0^2)^2 t^2} - y_1(t_i)) \\ &+ P(y_1(t_i), \epsilon_0\sqrt{1 + (\hbar/2m\sigma_0^2)^2 t^2} + y_1(t_i))] = 1, \end{aligned} \tag{36}$$

where $t = D/u_x$ is a constant. Note that, the first and second δ functions come from path determinations based on equations (28) and (33), respectively. In addition, the third and fourth δ functions are due to two distinguishable particles. If all times t_i in the last equation is taken to be t, then the summation on i can be changed to an integral over all paths that cross the screen S_2 at that time. Then, one can consider the probability of detecting two particles at two arbitrary points y_M and y_N

$$P_{12}(y_M, y_N) = \int_{y_M}^{y_M+\Delta} dy_1 \int_{y_N}^{y_N+\Delta} dy_2 P(y_1(t), y_2(t)), \tag{37}$$

which is similar to the prediction of SQM, but obtained in a Bohmian way. Thus, it appears that for such conditions, the possibility of distinguishing the two theories at the statistical level is denied, as was expected [1–4, 13].

But, we try to do our experiment in the following fashion: we record only those particles which are detected on the two sides of the x–axis simultaneously. That is, we eliminate the cases of detecting only one particle or when the pairs pass through the same slit, which means that we consider a selective detection of the particles. Furthermore, we assume that $y_0 = \delta \leq \sigma_0$, $\delta \ll Y$ and $\hbar t/2m\sigma_0^2 \gg 1$. Then, as we said earlier, the x–axis will not be an axis of symmetry and we have a new point on the S_2 screen along y-axis around which all pairs of particles will be detected symmetrically. Thus, based on BQM, there will be a length $L = 2y(t)$ on the S_2 screen where no particle is recorded, as shown in Fig. 2. On the other hand, based on SQM we have two alternatives:

i) The probability relation (18) is still valid and there is only a reduction in the intensity.

ii) SQM is silent about our selective detection.

In the first case, there is disagreement between the predictions of SQM and BQM. In the second case, BQM has a better predictive power, even at the statistical level. Of course, if y_0 varies randomly, then again the distinction between SQM and BQM is possible neither at the statistical nor at the individual level. Similar deviation from the axis of symmetry was also studied by Dewdney et al. [10]. Although their Gedanken experiment predicts inconsistent results between SQM and BQM under certain circumstance, but they obtain the same statistical results as SQM.

Now, let us see under what conditions there is a possibility for the existence of such an observable empty interval. If one considers L as an empty interval, then using equation (28), one must have

$$\frac{\hbar t y_0}{2m\sigma_0^2} \geq \frac{L}{2} \tag{38}$$

On the other hand, we have a limitation on the choice of y_0, because

$$\lim_{t\to\infty} \dot{y} = \frac{\hbar y_0}{2m\sigma_0^2}. \tag{39}$$

To secure the validity of the non–relativistic limit one must have $\dot{y} \leq 0.1c$. Thus, if we take $\sigma_0 = 10^{-7}m$ and $m_{min} = 9.1 \times 10^{-31}kg$, then $\hbar y_0/2m\sigma_0^2 \ll 0.1c$ leads to $y_0 \ll 5 \times 10^{-3}m$, which is consistent with the condition $y_0 \leq \sigma_0$, that was used for the preservation of the symmetry in the two slit system. If we assume that $y_0 = 10^{-9}m$, $u_x = 1m/s$, $D = 1m$, $\sigma_0 = 10^{-7}m$ and $L \geq 4cm$, then we get $m < 0.15GeV$. At first sight, it seems if we could use sources which emit pairs of π^0 or $\pi^{\pm}$ mesons with $M_{\pi^0} = 135MeV$ and $M_{\pi^{\pm}} = 140MeV$, simultaneously, the possibility of performing the aforementioned experiment would be provided. Unfortunately, the mean life time(τ) of π^0 and $\pi^{\pm}$ mesons is very small i. e. $8.4 \times 10^{-17}s$ and $2.6 \times 10^{-8}s$ respectively. Thus, under above mentioned conditions, the observation of the empty interval is impossible. However, if we take $y_0 = 10^{-9}m$, $\sigma_0 = 2.6 \times 10^{-9}m$, $D = 0.5m$, $u_x = 0.1c$ and $L \geq 10^{-6}m$, we get $m < 150MeV$ and $\pi^{\pm}$ is suitable for this experiment. It is worthy to note that, although the distance between any two neighboring maxima on the screen S_2 is not quite given by the classical formula $\delta y = \lambda D/2Y$, quoted in elementary optics [3], but we can estimate $\delta y \sim 0.4\mu m$ with $Y = 5 \times 10^{-7}m$, which is comparable with the empty length L. Similarly for $K^{\pm}$ mesons ($M_{K^{\pm}} = 493.6MeV$, $\tau = 1.2 \times 10^{-8}s$), a suitable choice of parameters is $y_0 = 10^{-9}m$, $\sigma_0 = 1.4 \times 10^{-9}m$, $D = 0.3m$, $u_x = 0.1c$ and $L = 10^{-6}m$. In this case we have $\delta y \sim 0.1\mu m$.

On the other hand, we can consider $y_0 = 10^{-8}m$, $\sigma_0 = 10^{-7}m$, $D = 0.5m$, $u_x = 80m/s$ and $L \geq 10^{-4}m$. Then we obtain $m \leq 3.7GeV$ and we have $\delta y \sim 6 \times 10^{-4}m$ which is again comparable with L. The only known bosonic particle that can satisfy above conditions with a large mean life is α particle. In addition, based on the relation (35) and above considrations we have $\epsilon_{max} = 1.3, 0.4\mu m$ and $0.5mm$ for $\pi^{\pm}, K^{\pm}$ and α particles, respectively. Thus, although it seems that performing such experiments is very hard but it is possible to provide suitable conditions for the detection of observable differences between the two theories, particularly at the statistical level.

5 Conclusion

We noticed that in a special two slit experiment in which two bosonic particles are emitted from a source S_1 simultaneously, by making use of Gaussian wave packets and the symmetry of the wave function and the symmetry of the apparatus, it is possible to predict the y component of the center of mass of the system in terms of the y component of that point at $t = 0$, the mass of the particles and the half–width of the slits. If $y_0 = 0$ or $y_0 = \delta \leq \sigma_0$, $\delta \ll Y$ and the conditions are chosen such that $\hbar D/2m\sigma_0^2 u_x \ll 1$, then all detections around the x–axis will be symmetrical. Furthermore, two particles which pass through one slit will be detected simultaneously on the same side of the x–axis. Thus, the prediction of BQM are inconsistent with those of standard interpretation only when the simultaneous detection of each pairs of particles is under consideration. But, if we observe the pattern resulting from the detection of all pairs of particles, then the two theories agree, as was expected. In addition, if $y_0 = \delta \leq \sigma_0$ and $\delta \ll Y$ but $\hbar D/2m\sigma_0^2 u_x \gg 1$, then only a single detection on the two sides of the x–axis is enough to predict the y–component of the center of mass of all subsequent particles, and all detections around this point will be symmetrical. On the other hand, since in BQM the particles are distinguishable and their past history are known, then by using a selective detection of the particles, one can have predictions which are inconsistent with the SQM or predictions for which the SQM is silent. If we eliminate all cases of one–particle detection and all cases of two–particle detection on the one side of the x–axis, then by adjusting y_0 and satisfying the condition $y_0\hbar D/m\sigma_0^2 u_x \geq L$, one can have a region of the size L or larger on the screen in which no particle is detected. Thus, not only in the case of simultaneous detection of the two particles ,at the individual level, we have discrepancy with the SQM, even when all detected particles are considered, in a selective detection process, we have a region with no particle detection-an empty region not predicted by SQM. Therefore, this experiment seems to shed light on the question of whether wave function provides a complete description of a system, and whether Bohmian position is an actual position or it is simply a mathematically concept.

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Figure Caption

Figure 1. A two-slit experiment in which two identical bosonic particles are emitted from the source S_1, then they pass through slits A, B, and finally they are detected on the screen S_2, simultaneously. We assume that $y_0 = 0$ or $y_0 = \delta \leq \sigma_0$ and $\hbar t/2m\sigma_0^2 \ll 1$. It is clear that dashed lines are not real trajectories.

Figure 2. The same two-slit experiment in which $y_0 = \delta \leq \sigma_0$, $\hbar t/2m\sigma_0^2 \gg 1$, and selective detection is considered. All detections are symmetric on the two sides of y_{cm} on the screen S_2. Thus, L shows the empty interval in the final observed pattern. Dotted doshed lines are not real trajectories.